Differential Equations Seminar: Week 2 Solutions

1. $\mathbf{x} = \mathbf{0}$

2.

(a) $\lambda = 1 \pm 2i$, $\mathbf{v} = \begin{bmatrix} \frac{1}{2} \pm \frac{i}{2} \\ 1 \end{bmatrix}$

(b) $\lambda_1 = -3$, $\lambda_2 = -1$, $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $\lambda_1, 2 = 1 \pm 2i$, $\lambda_3 = 1$, $\mathbf{v}_1, 2 = \begin{bmatrix} 0 \\ \pm i \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$

3.

(a) $\mathbf{x}(t) = c_1 e^{(1+2i)t} \begin{bmatrix} \frac{1}{2} + \frac{i}{2} \\ 1 \end{bmatrix} + c_2 e^{(1-2i)t} \begin{bmatrix} \frac{1}{2} - \frac{i}{2} \\ 1 \end{bmatrix}$ which is equivalent to
$\mathbf{x}(t) = c_1 e^t \begin{bmatrix} \frac{1}{2}\cos(2t) - \frac{1}{2}\sin(2t) \\ \cos(2t) \end{bmatrix} + c_2 e^t \begin{bmatrix} \frac{1}{2}\cos(2t) + \frac{1}{2}\sin(2t) \\ \sin(2t) \end{bmatrix}$

(b) $\mathbf{x}(t) = c_1 e^{-3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

4. Since λ is an eigenvalue of A if and only if $\det(A - \lambda I) = 0$, $\lambda = 0$ is an eigenvalue of A if and only if $\det(A) = 0$.