## Differential Equations Seminar: Week 2 Exercises

1. Solve the matrix equation  $A\mathbf{x} = \mathbf{0}$ , where

$$A = \left[ \begin{array}{rrrr} 1 & 0 & -1 \\ 3 & 1 & 1 \\ -1 & 1 & 2 \end{array} \right].$$

2. Find all eigenvalues and eigenvectors of the given matrices.

(a) 
$$\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$
  
(b)  $\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$   
(c)  $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix}$ 

3. Find the general solution of the given system of equations.

(a) 
$$\mathbf{x}' = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \mathbf{x}$$
  
(b)  $\mathbf{x}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \mathbf{x}$ 

4. Prove that  $\lambda = 0$  is an eigenvalue of A if and only if A is singular. (Hint: det A = 0 if and only if A is singular.)