## Differential Equations Seminar: Week 2 Exercises

1. Solve the matrix equation $A \mathbf{x}=\mathbf{0}$, where

$$
A=\left[\begin{array}{ccc}
1 & 0 & -1 \\
3 & 1 & 1 \\
-1 & 1 & 2
\end{array}\right]
$$

2. Find all eigenvalues and eigenvectors of the given matrices.
(a) $\left[\begin{array}{ll}3 & -2 \\ 4 & -1\end{array}\right]$
(b) $\left[\begin{array}{cc}-2 & 1 \\ 1 & -2\end{array}\right]$
(c) $\left[\begin{array}{ccc}1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1\end{array}\right]$
3. Find the general solution of the given system of equations.
(a) $\mathbf{x}^{\prime}=\left[\begin{array}{ll}3 & -2 \\ 4 & -1\end{array}\right] \mathbf{x}$
(b) $\mathbf{x}^{\prime}=\left[\begin{array}{cc}-2 & 1 \\ 1 & -2\end{array}\right] \mathbf{x}$
4. Prove that $\lambda=0$ is an eigenvalue of $A$ if and only if $A$ is singular. (Hint: $\operatorname{det} A=0$ if and only if $A$ is singular.)
